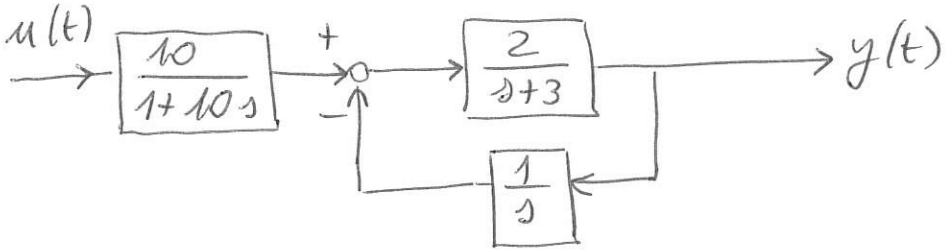
Si consideri il sistema



a) Si traccino i diagrammi di Bode approssimati di modulo e fase.

b) Si dica quanto vale approssimativamente l'uscita a regime se $u(t) = -5 + 20\cos(t)$. Si confronti il risultato ottenuto tramite il calcolo esatto.

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a) $$G(s) = \frac{10}{1+10s} \cdot \frac{\frac{2}{s+3}}{1+\frac{1}{s}\cdot\frac{2}{s+3}} = \frac{10}{1+10s} \cdot \frac{2s}{s^2+3s+2}$$

$$\Rightarrow G(s) = \frac{20\,s}{(1+10s)(2+s)(1+s)}$$

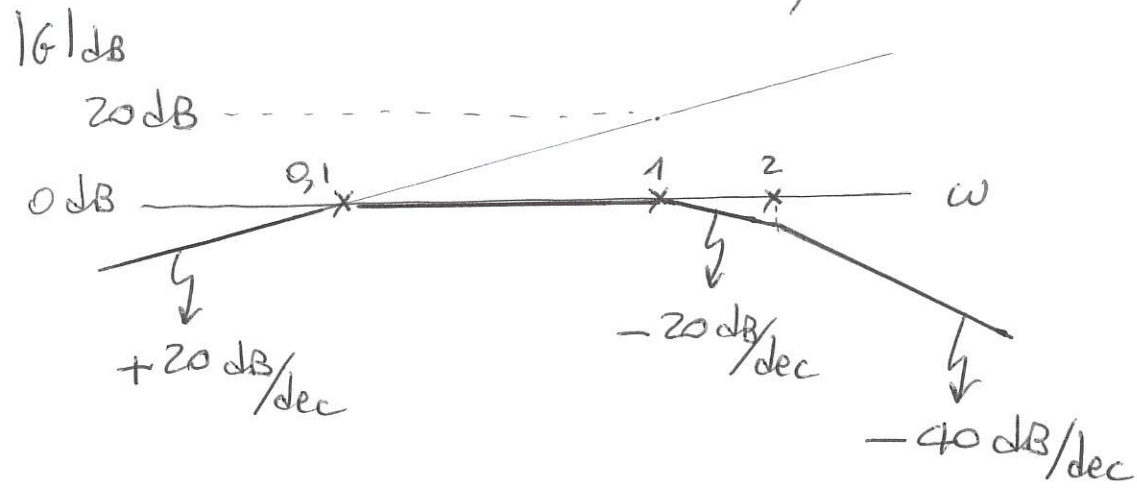
In forma standard

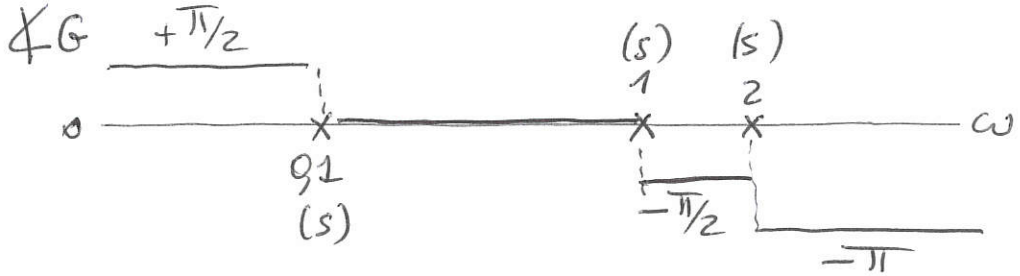
$$G(s) = \frac{10\,s}{(1+10s)\left(1+\frac{1}{2}s\right)(1+s)}$$

- $\mu = 10 \rightarrow \mu_{dB} = 20\,dB$ e $\measuredangle \mu = 0$
- zero nell'origine
- |poli| in 0,1 ; 1 ; 2 tutti stabili
- Alle basse frequenze $\rightarrow 10s$ $\rightsquigarrow$ Bode modulo: retta con pendenza $+20\,dB/dec$ che in $\omega = 1$ vale $20\,dB$

$\rightarrow 10\,s \rightsquigarrow$ <u>Bode fase</u>

$\measuredangle \mu + \measuredangle i\omega = 0 + \pi/2 = \pi/2$





b) $y(t) = -5 \cdot G(0) + 20\,|G(i)| \cos(t + \measuredangle G(i))$

$G(0) = 0$

<u>Bode approssimato</u> $\rightsquigarrow |G(i)|_{dB} = 0\,dB$

$\hookrightarrow |G(i)| = 1$

$\measuredangle G(i) = -\pi/4$

$\Rightarrow y(t) = 20 \cos(t - \pi/4)$

<u>Calcolo esatto</u> $\rightsquigarrow |G(i)| = \dfrac{10\,|i|}{|1+10i|\,|1+\frac{1}{2}i|\,|1+i|} =$

$= \dfrac{10}{\sqrt{101}\,\sqrt{\frac{5}{4}}\,\sqrt{2}} \simeq 0{,}63$

$$\measuredangle G(i) = \measuredangle(10i) - \measuredangle(1+10i) - \measuredangle\left(1+\tfrac{1}{2}i\right) - \measuredangle(1+i) =$$

$$= \pi/2 - \underbrace{\operatorname{arctg} 10}_{\sim \pi/2} - \underbrace{\operatorname{arctg} \tfrac{1}{2}}_{\sim \pi/6} - \underbrace{\operatorname{arctg} 1}_{\pi/4}$$

$$\Rightarrow \measuredangle G(i) \simeq -\frac{5}{12}\pi$$

$$\Rightarrow y(t) = 12{,}6 \cos\left(t - \frac{5}{12}\pi\right)$$

<u>NOTE</u> $|G(i)|$ approssimato $\neq |G(i)|$ esatto perché in $\omega = 1$ ho $|\text{polo}| \rightsquigarrow$ max errore nell'approssimazione di Bode

$\measuredangle G(i)$ esatta $\neq -\pi/4$ perché in $\omega = 1$ la fase risente della perdita di fase del vicino polo stabile in $\omega = 2$

Si consideri il sistema descritto dalla funzione di trasferimento

$$G(s) = \frac{10}{s} \frac{s+91}{s^2+s+100}$$

a) Determinare graficamente la risposta all'impulso.

b) Tracciare i diagrammi di Bode di modulo e fase.

c) Supponendo che l'ingresso applicato al sistema sia $u(t) = 2\,\text{sen}(10t)$, determinare, in modo esatto e approssimato, l'ampiezza dell'uscita sinusoidale. Si giustifichi il risultato ottenuto.

In forma standard $G(s) = \frac{0{,}01}{s} \cdot (1+10s) \frac{100}{s^2+s+100}$

a) $G = \frac{1}{s} \cdot \tilde{G}$ con $\tilde{G}(s) = 0{,}01(1+10s) \cdot \frac{100}{s^2+s+100}$

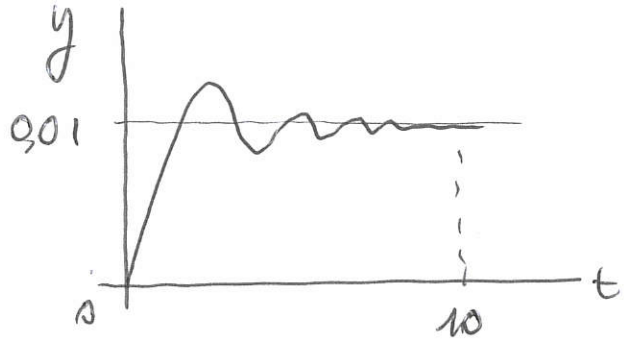
Risposta a impulso di G = Risposta a scalino di $\tilde{G}$

$p_{1,2} = \frac{-1 \pm i\sqrt{399}}{2} \rightarrow \exists\, \infty$ oscillazioni

$\mathrm{Re}(p_D) = -\frac{1}{2}$ $\quad T_D = 2$ $\quad T_R = 10$

$y_\infty = \tilde{G}(0) = 0{,}01$

$r = 1 \rightarrow y(0) = 0$ $\quad \dot{y}(0) = 0{,}01 \cdot 10 \cdot 100 = 10 > 0$



b) polo nell'origine

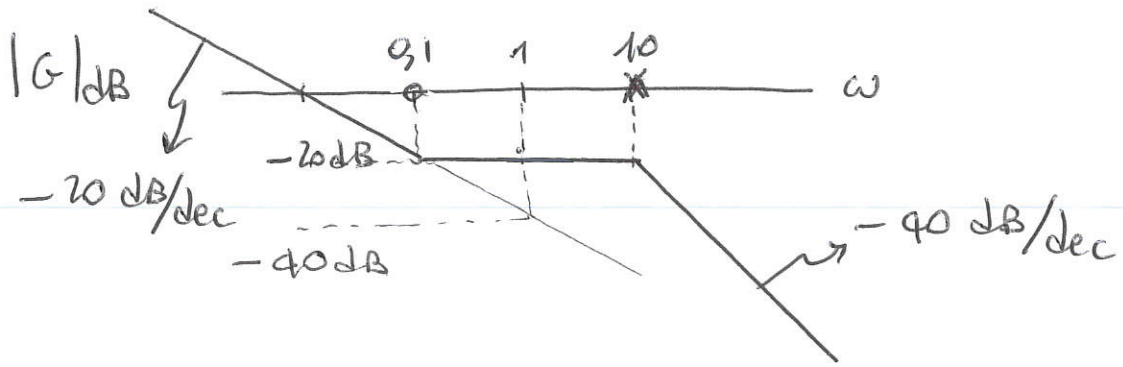
- $\mu = 0{,}01 \rightarrow \mu_{dB} = -40\ dB$
- Basse frequenze $\rightarrow |G|_{dB}$ = retta con pendenza $-20\ dB/dec$ che in $\omega = 1$ assume valore $\mu_{dB} = -40\ dB$

$$\rightarrow \measuredangle G = \measuredangle \mu - \pi/2 = -\pi/2$$

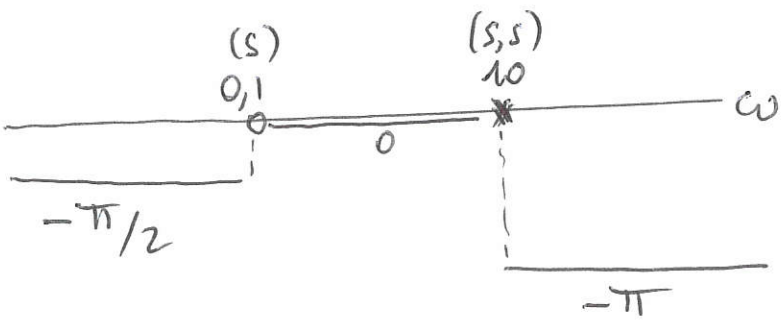
- poli $\mathbb{C}$ con $\omega_n = 10$ (stabili)

NOTA: $2\xi\omega_n = 1 \rightarrow \xi = \frac{1}{20} = 0{,}05 \rightarrow$ marcata risonanza in $\omega_n = 10$

- |zero| in 0,1 stabile



$\measuredangle G$



c) $Y = |G(i\omega)|\, U \rightarrow Y = |G(10i)| \cdot 2$

APPROX $\rightarrow$ Bode approssimato

$$|G(10i)|_{dB} = -20\,dB \rightarrow |G(10i)|^{APPROX} = 0{,}1$$

$$Y^{APPROX} = 0{,}1 \cdot 2 = 0{,}2$$

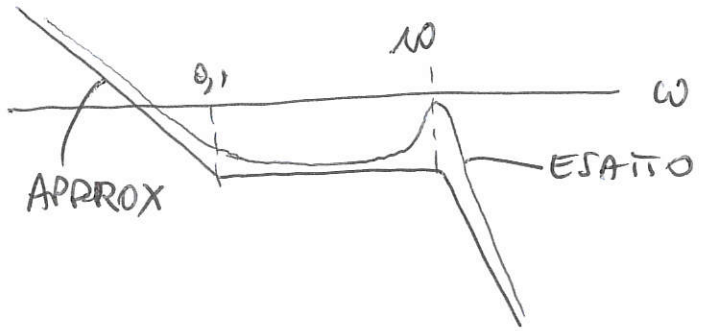
ESATTO

$$|G(10i)| = \frac{10}{|10i|} \frac{|10 + 0{,}1i|}{|10i|} \simeq \frac{10}{10} \cdot \frac{10}{10} = 1$$

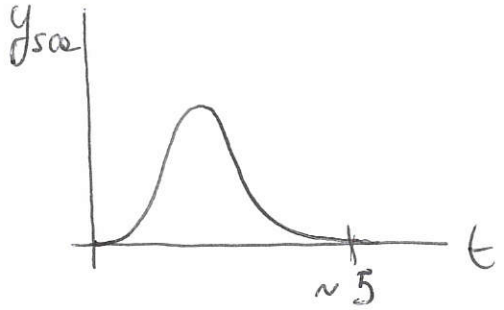
$$Y^{ESATTO} = 1 \cdot 2 = 2$$

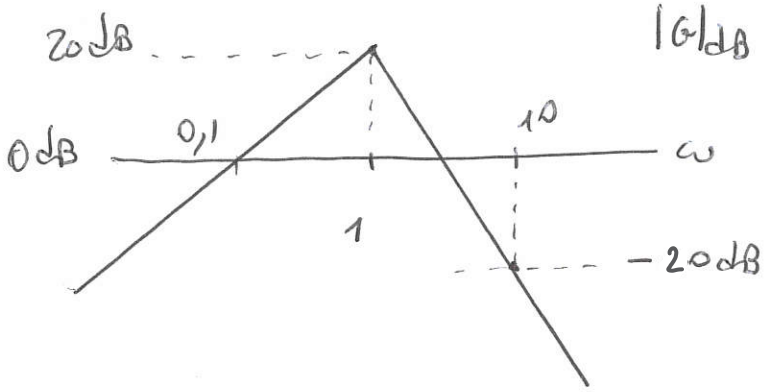
$Y^{APPROX} \neq Y^{ESATTO}$ molto diverso!

Ciò è dovuto al fatto che in $\omega = 10 = \omega_n \simeq \omega_r$ il (pulsazione di risonanza) sistema ha una risonanza marcata, così che, in $\omega \simeq \omega_n$, $|G(i\omega)|_{dB}^{ESATTO}$ è molto diverso da $|G(i\omega)|_{dB}^{APPROX}$



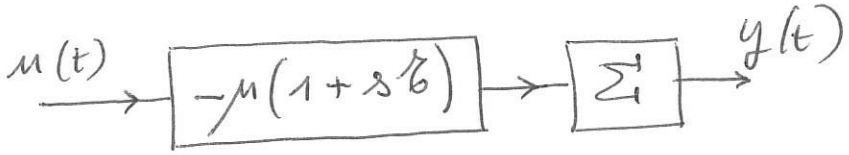
Un sistema Σ ha risposta allo scalino e diagramma di Bode approssimato del modulo rappresentati in figura:



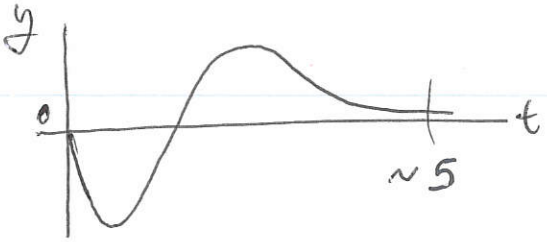


a) Determinare una possibile funzione di trasferimento per Σ compatibile con i due diagrammi

b) Il sistema Σ viene poi collegato in cascata con un nuovo sistema, ottenendo



Determinare $\mu > 0$ e τ tale che il sistema aggregato abbia risposta a scalino unitario mostrata in figura



$\dot{y}(0) = -1$

c) Tracciare i diagrammi di Bode di modulo e fase del sistema aggregato

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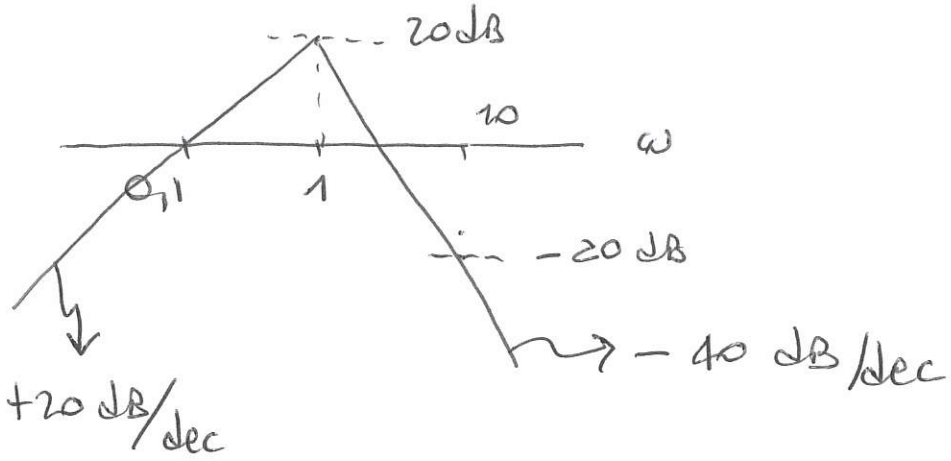
a) y_{sca} limitata $\rightarrow$ ES $\rightarrow$ poli stabili
$\nexists\, \infty$ oscill $\rightarrow$ poli $\mathbb{R}$
$T_R = 5 \rightarrow \mathrm{Re}(p_s) = -1$

$y_\infty = 0 \rightarrow G(0) = 0 \rightarrow$ zero nell'origine



in $\omega = 1 \rightarrow \mu_{dB} = 20\,dB$

$\mu = 10$

In $\omega = 1$ la pendenza varia di -60 dB/dec (da $+20$ a -40) $\Rightarrow$ ho 3 poli

$$G(s) = \frac{10\,s}{(1+s)^3} \rightsquigarrow r = 2, \text{ compatibile con } y(0) = \dot{y}(0) = 0, \quad \ddot{y}(0) = 10 > 0$$

b) $$\tilde{G} = -\mu(1+s\tau)G = -\mu(1+s\tau)\frac{10\,s}{(1+s)^3}$$

$$\dot{y}(0) = -1 = -10\mu\tau \rightarrow 10\mu\tau = 1$$

$\mu > 0 \Rightarrow \tau > 0$ zero stabile

$N = 2 \Rightarrow m_s = 2$ ($m_s = 1 \Rightarrow$ # dispari di estremi)

$\delta = 0$

Per esempio posso porre lo zero stabile in $\frac{1}{10}$ $(\tau = 10)$

$m_s = 2$
$\delta = 0$ $\rightarrow N = 2$

$10 \mu \tau = 1 \Rightarrow \mu = 0{,}01$

Pertanto $\tilde{G} = -\dfrac{0{,}1\,(1+10s)\,s}{(1+s)^3}$

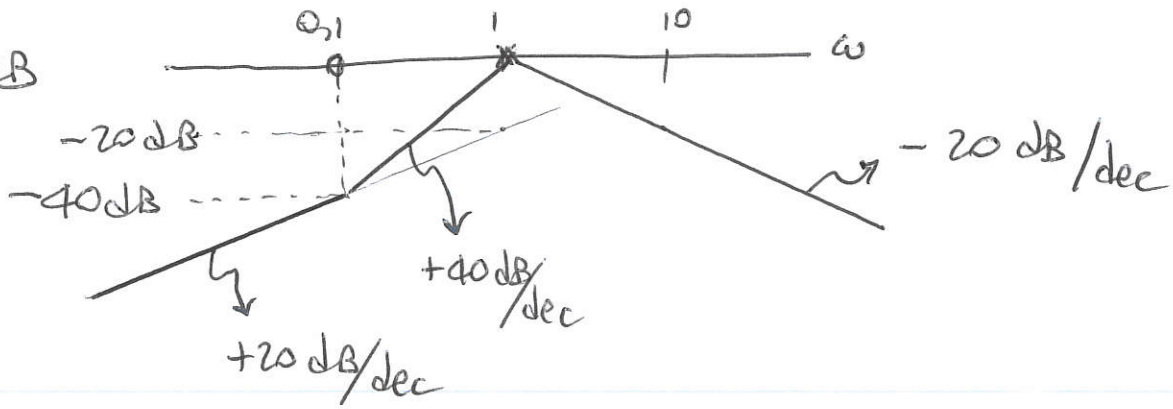
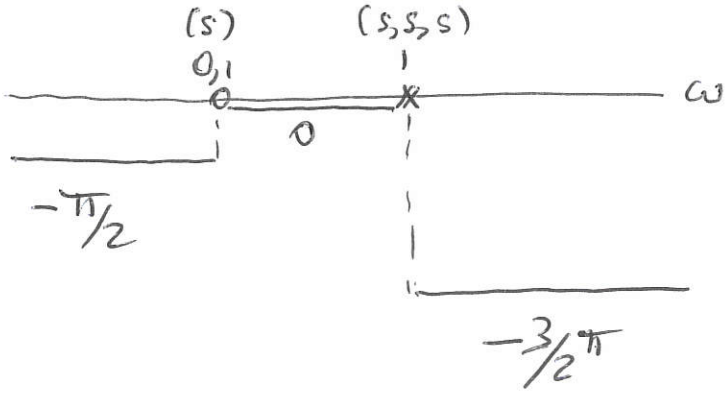
c) $\tilde{\mu} = 0{,}1 \rightarrow \tilde{\mu}_{dB} = -20\ dB$

1 zero nell'origine

|poli| in 1, 1, 1 stabili

|zero| in 0,1 stabile

$|\tilde{G}|_{dB}$



$\measuredangle \tilde{G}$